

Heterotic Vacua from general (non-) Abelian Bundles

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We report on the construction of four-dimensional string vacua by considering general abelian and non-abelian bundles on an internal Calabi-Yau for both heterotic theories. The structure of the resulting gauge sector is extremely rich and gives rise to many new model building possibilities. We analyse the chiral spectrum including the contribution from heterotic five-branes and provide the general consistency conditions. The one-loop corrected supersymmetry condition on the bundles is found to be that of π -stability. As an application we present a supersymmetric Standard-Model like example for the $SO(32)$ string with $U(n)$ bundles on an elliptically fibered Calabi-Yau.

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1 Introduction and summary

Even if we are still far from a complete understanding of all the mysteries string theory confronts us with, it is widely accepted by now that the theory gives rise to an overwhelmingly large number of at least meta-stable vacua. Each of them can be approximately described by a low-energy effective theory which, unlike adhoc phenomenologically motivated guesses of effective lagrangians, is guaranteed to possess a consistent ultra-violet completion. A thorough analysis of the set of possible string vacua is therefore an equally challenging and indispensable endeavour. The present state of the art only admits the description of a tiny fraction of possible constructions, the mathematics of the emergent structure being simply too intricate in most cases. One large class of solvable string compactifications with phenomenologically appealing features is given by type IIA toroidal orientifolds with intersecting $D6$ -branes (for the most recent review see [1]). In view of the intense model building attempts in this framework, it would be very desirable to extend this type of models to the generic case of curved internal background manifolds. Progress in this direction is severely hampered by our lacking knowledge of general supersymmetric cycles on which the branes wrap since the special Lagrangian condition, being non-holomorphic in nature, is of course beyond the power of complex geometry. On the other hand, mirror symmetry maps intersecting branes to B-type branes on holomorphic cycles carrying *abelian* gauge instantons. The sLag condition gets replaced by a stability constraint on the holomorphic gauge bundle. An immediate two-fold extension of the intersecting branes picture is therefore to consider *general* $U(n)$ bundles on $D9$ -branes wrapping a *non-trivial* Calabi-Yau. This setup is in turn reminiscent of the S-dual heterotic string constructions. Here, by contrast, most attempts in the literature have focused on the embedding of non-abelian vector bundles with vanishing first Chern classes into $E_8 \times E_8$ ¹.

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¹ For some early examples with $U(n)$ bundles see [10–13] and further references in [2].

In this talk we would like to report on recent activities in the exploration of four-dimensional vacua in both heterotic theories defined by allowing also for line and $U(n)$ bundles. In the $E_8 \times E_8$ case the use of line bundles yields a large number of new scenarios of breaking E_8 down to GUT groups or further to the Standard Model group *without the need of non-trivial Wilson lines on the Calabi-Yau* [4]. For the $SO(32)$ string [2, 5] we will systematically give a class of $U(n)$ embeddings into $SO(32)$ which is the direct S-dual of magnetized $D9$ -branes with non-abelian bundles [3]. We present the resulting chiral spectrum, taking also into account the possibility of allowing for additional heterotic five-branes. The absence of global and local anomalies imposes a number of non-trivial consistency conditions on the topological data. Besides being relevant for determining the unbroken abelian part of the four-dimensional gauge group, the one-loop GS counterterms allow for a derivation of the corrections to the supersymmetry condition on the bundles at one loop. These arise as a D-term in the supergravity description. The result is that the perturbatively exact supersymmetry constraint is that of π -stability [8], which, in the S-dual picture, constitutes the perturbative part of the well-known Π -stability condition. As an illustration of the rich model building possibilities behind these constructions we finally give an example in the $SO(32)$ framework by means of the spectral cover construction of stable vector bundles on an elliptically fibered Calabi-Yau. Many more of such phenomenologically interesting models are to be discovered. In particular, vacuum search in the direct Standard Model group breaking scenarios of [4] on manifolds with trivial fundamental group might turn out fruitful. Lack of space regrettably forces us to refer the reader to the original papers [2–5] for more detailed information and for a more complete list of relevant references.

2 General embeddings and their chiral spectrum

Four-dimensional heterotic compactifications are, at the perturbative level, specified by a vector bundle W over the internal Calabi-Yau manifold \mathcal{M} together with an embedding of its structure group G into $E_8 \times E_8$ or $Spin(32)/\mathbb{Z}_2^2$, respectively. By standard arguments, giving VEVs to the field strengths of G on the internal manifold breaks the four-dimensional gauge group to the commutator of G in $E_8 \times E_8$ or $SO(32)$. In general we will be interested in Whitney sums of bundles $W = \bigoplus_s V_s$ with structure group $\prod_s G_s$.

Beyond the hitherto mostly studied case where all summand bundles are semi-simple, the $E_8 \times E_8$ string naturally allows for the embedding of sums of $SU(n_i)$ vector bundles together with non-trivial line bundles, i.e. $W = \bigoplus_{i=1}^K V_i \oplus \bigoplus_{m=1}^M L_m$, or modifications thereof³, as detailed in [4]. The concrete form of the four-dimensional gauge group depends very much on the details of the chosen embedding. In particular, concerning its abelian part, only the massless combinations of $U(1)$ factors survive as a gauge symmetry.

For the $SO(32)$ theory a large class of models is defined by taking the structure group of V_i to be $U(n_i)$ and diagonally embedding it into a $U(M_i)$ gauge factor with $M_i = n_i N_i$, $i \in \{1, \dots, K\}$ [2]. This breaks the original $SO(32)$ gauge group to $SO(2M) \times \prod_{i=1}^K U(N_i)$ with $M + \sum_{i=1}^K N_i = 16$, again up to the issue of some subgroup of $U(1)^K$ potentially becoming massive. The subsequent notation will be referring to these two types of constructions in both heterotic theories.

To determine the massless spectrum, one analyses the splitting of the adjoint representation of $E_8 \times E_8$ or $SO(32)$ into various irreducible representations $(R_j; r_j)$ under the four-dimensional group and the internal one. Thanks to the non-trivial internal gauge background we find four-dimensional *chiral* matter in representations R_j specified by the cohomology class $H^*(\mathcal{M}, U_j)$, where, loosely speaking, the bundle U_j is determined as the one with fundamental representation r_j . In particular, the Hirzebruch-Riemann-Roch index theorem computes the net number of four-dimensional chiral fermions in the representation

² In standard abuse of notation we will refer to this case as $SO(32)$ though the \mathbb{Z}_2 projection is actually a different one.

³ One can also take the V_i to be $U(n_i)$ -bundles and adjust the $c_1(L_m)$ to yield $c_1(W) = 0$.

R_j as

$$\chi(\mathcal{M}, U_j) = \sum_{n=0}^3 (-1)^n \dim H^n(\mathcal{M}, U_j) = \int_{\mathcal{M}} \left[\text{ch}_3(U_j) + \frac{1}{12} c_2(T) c_1(U_j) \right]. \quad (1)$$

Concretely, for the class of $SO(32)$ models we arrive at the perturbative massless spectrum

$$496 \rightarrow \left(\begin{array}{c} (\mathbf{Anti}_{SO(2M)})_0 \\ \sum_{j=1}^K (\mathbf{Adj}_{U(N_j)}; \mathbf{Adj}_{U(n_j)}) \\ \sum_{j=1}^K (\mathbf{Anti}_{U(N_j)}; \mathbf{Sym}_{U(n_j)}) + (\mathbf{Sym}_{U(N_j)}; \mathbf{Anti}_{U(n_j)}) + h.c. \\ \sum_{i < j} (\mathbf{N}_i, \mathbf{N}_j; \mathbf{n}_i, \mathbf{n}_j) + (\mathbf{N}_i, \bar{\mathbf{N}}_j, \mathbf{n}_i, \bar{\mathbf{n}}_j) + h.c. \\ \sum_{j=1}^K (2M, \mathbf{N}_j; \mathbf{n}_j) + h.c. \end{array} \right). \quad (2)$$

For examples in the context of the $E_8 \times E_8$ string we refer the reader to [4].

In addition to this perturbative sector, both heterotic theories are well-known to comprise five-branes H5. For four-dimensional Lorentz invariance, they are taken to be spacetime-filling and wrap internal 2-cycles, which in supersymmetric configurations have to be holomorphic. In the E_8 theory, the worldvolume of the five-brane contains a tensor field and does not yield any additional charged chiral matter in four dimensions [14] in agreement with the observation that in Horava-Witten theory, the corresponding M5-brane can be pulled into the 11D bulk. We will therefore focus in the subsequent discussion on the $SO(32)$ case. Here the worldvolume of a H5-brane supports a massless gauge field, which compared to the $E_8 \times E_8$ H5-brane leads to different low energy physics. The gauge group can be deduced by noting that S-duality directly maps the H5 to the D5-brane in Type I [15], which is known to give rise to symplectic gauge groups. More precisely, a brane wrapping the holomorphic curve $\Gamma = \sum_a N_a \Gamma_a$, $N_a \in \mathbb{Z}_0^+$ yields a gauge group factor $\prod_a Sp(2N_a)$, where the enhancement is due to the multiple wrapping around each irreducible curve Γ_a . The above decomposition of Γ may not be unique and the gauge group may therefore vary in the different regions of the associated moduli space. However, its total rank and the total number of chiral degrees of freedom charged under the symplectic groups are only dependent on Γ , of course.

The cancellation of gravitational anomalies on the $SO(32)$ H5-brane requires a Chern-Simons like coupling of the H5-brane to the bulk by anomalous inflow arguments. Inspired further by heterotic-Type I duality, one can infer that the effective low energy action on the H5-branes contains a piece

$$S_{H5_a} = -\mu_5 \int_{\mathbb{R}_{1,3} \times \Gamma_a} \sum_{n=0}^1 B^{4n+2} \wedge \left(N_a + \frac{\ell_s^4}{4(2\pi)^2} \text{tr}_{Sp(2N_a)} F_a^2 \right) \wedge \frac{\sqrt{\hat{\mathcal{A}}(\text{T}\Gamma_a)}}{\sqrt{\hat{\mathcal{A}}(\text{N}\Gamma_a)}}, \quad (3)$$

with the H5-brane tension $\mu_5 = \frac{1}{(2\pi)^5 (\alpha')^3}$. $\text{T}\Gamma_a$ and $\text{N}\Gamma_a$ denote the tangent bundle and the normal bundle, respectively, of the 2-cycle Γ_a , which for concreteness we take to be irreducible from now on and wrapped by a stack of N_a H5-branes. The curvature occurring in the definition of the $\hat{\mathcal{A}}$ genus is defined as $\mathcal{R} = -i\ell_s^2 R$ ($\ell_s \equiv 2\pi\sqrt{\alpha'}$). Note that the universal five-brane coupling to $B^{(6)}$ (defined by $*_{10} dB^{(2)} = e^{2\phi_{10}} dB^{(6)}$) must also be present for the $E_8 \times E_8$ theory irrespective of the concrete form of the further terms in its worldvolume action.

To make contact with the vector bundle theoretic discussion of the massless spectrum, it is useful to describe the $SO(32)$ H5-brane wrapping Γ_a as the skyscraper sheaf $\mathcal{O}|_{\Gamma_a}$ with topological invariants given by $\text{ch}(\mathcal{O}|_{\Gamma_a}) = (0, 0, -\gamma_a, 0)$.

As anticipated, the $SO(32)$ H5-branes give rise to chiral matter in the bifundamental $(\mathbf{N}_i, 2\mathbf{N}_a)_{1(i)}$, which is counted by the index

$$\chi(X, V_i \otimes \mathcal{O}|_{\Gamma_a}^*) = - \int_X c_1(V_i) \wedge \gamma_a. \quad (4)$$

Here γ_a denotes the Poincaré dual 4-form corresponding to the 2-cycle Γ_a .

To conclude this summary of the heterotic particle content, Table 1 exemplifies the chiral matter arising from the perturbative and non-perturbative sector of the $SO(32)$ theory with our favourite embedding (2).

reps.	$H = \prod_{i=1}^K SU(N_i) \times U(1)_i \times SO(2M) \times \prod_{a=1}^L Sp(2N_a)$
$(\mathbf{Adj}_{U(N_i)})_{0(i)}$	$H^*(X, V_i \otimes V_i^*)$
$(\mathbf{Sym}_{U(N_i)})_{2(i)}$	$H^*(X, \bigwedge^2 V_i)$
$(\mathbf{Anti}_{U(N_i)})_{2(i)}$	$H^*(X, \bigotimes_s^2 V_i)$
$(\mathbf{N}_i, \mathbf{N}_j)_{1(i), 1(j)}$	$H^*(X, V_i \otimes V_j)$
$(\mathbf{N}_i, \overline{\mathbf{N}}_j)_{1(i), -1(j)}$	$H^*(X, V_i \otimes V_j^*)$
$(\mathbf{Adj}_{SO(2M)})$	$H^*(X, \mathcal{O})$
$(2\mathbf{M}, \mathbf{N}_i)_{1(i)}$	$H^*(X, V_i)$
$(\mathbf{Anti}_{Sp(2N_a)})$	$\text{Ext}_X^*(\mathcal{O} _{\Gamma_a}, \mathcal{O} _{\Gamma_a})$
$(\mathbf{N}_i, 2\mathbf{N}_a)_{1(i)}$	$\text{Ext}_X^*(V_i, \mathcal{O} _{\Gamma_a})$
$(2\mathbf{N}_a, 2\mathbf{N}_b)$	$\text{Ext}_X^*(\mathcal{O} _{\Gamma_a}, \mathcal{O} _{\Gamma_b})$

Table 1 Massless spectrum of the $SO(32)$ theory with the structure group taken to be $G = \prod_{i=1}^K U(n_i)$.

3 Anomalies, tadpoles and massive $U(1)$ factors

In order to describe a well-defined string compactification the bundle data has to satisfy a number of non-trivial consistency conditions. A powerful guideline in their identification is to search for the appearance of possible global or local anomalies on the worldsheet or in the effective supergravity.

To begin with, the absence of anomalies in the two-dimensional non-linear sigma model [9] requires

$$c_1(W) \in H^2(\mathcal{M}, 2\mathbb{Z}). \quad (5)$$

In the $SO(32)$ theory, this constraint ensures that the number of chiral fermions in the fundamental of the $Sp(2N_a)$ groups be even, as is obvious from (4), and therefore has a simple interpretation as the condition for the vanishing of a global $Sp(2N_a)$ Witten anomaly on every probe brane.

Let us turn our attention to the local anomalies of the spacetime effective theory. It is well-known that for absence of anomalies in ten dimensions the string tree-level effective action has to be modified by two important contributions: First, the three-form field strength comprises not only gauge, but also essential gravitational CS terms, $H = dB^{(2)} - \frac{\alpha'}{4}(\omega_Y - \omega_L)$, which enter into the effective action via the usual kinetic term $S_{kin} = -\frac{1}{4\kappa_{10}^2} \int e^{-2\phi_{10}} H \wedge \star_{10} H$ with $\kappa_{10}^2 = \frac{1}{2}(2\pi)^7 (\alpha')^4$. We immediately see that the Bianchi identity for H_3 takes the form $dH_3 = \ell_s^2 \left(\frac{1}{4(2\pi)^2} [\text{tr} R^2 - \text{tr} F^2] + \sum_a N_a \gamma_a \right)$ after including also the CS coupling 3 of the dual field $B^{(6)}$ to the H5 brane. The traces are taken in the fundamental representation of $SO(1, 9)$ and $E_8 \times E_8$ or $SO(32)$, respectively. This translates into the cohomological condition

$$-c_2(T) + \sum_{a=1}^L N_a \gamma_a = \begin{cases} \sum_{i=1}^K \text{ch}_2(V_{n_i}) + \sum_{m=1}^M a_m c_1^2(L_m) & (E_8 \times E_8), \\ \sum_{i=1}^K N_i \text{ch}_2(V_i) & (SO(32)). \end{cases} \quad (6)$$

Note that in the first case the coefficients a_m depend on the concrete embedding and are determined as $4a_m = \text{tr}_{E_8} Q_m^2$. Besides its interpretation as a tadpole condition in supergravity, this constraint ensures, in the $SO(32)$ case, the absence of cubic non-abelian anomalies⁴.

⁴ Note, however, that in the $E_8 \times E_8$ embeddings of [4] these non-abelian anomalies vanish even without imposing (6). By contrast, in both heterotic theories it is required for the generalized Green-Schwarz mechanism to cancel all (possibly mixed) abelian anomalies

Since it will become relevant in the next section, we would like to draw the reader's attention to the fact that the crossterm $S_{kin} = \frac{1}{8\pi\ell_s^6} \int (\text{tr} F^2 - \text{tr} R^2) \wedge B^{(6)}$ contained in the kinetic action of H effectively appears at one loop in string perturbation theory.

The second one-loop piece of information provided by anomaly counterterms is of course the celebrated ten-dimensional Green-Schwarz term $S_{GS} = \frac{1}{48(2\pi)^3\ell_s^2} \int B^{(2)} \wedge X_8$ with the standard anomaly eight-form. As shown in great detail in [4, 5] dimensional reduction of the kinetic and GS-terms provides precisely the right counterterms to cancel all abelian anomalies arising in four dimensions. Rather than displaying all terms here, we would like to stress that one important ingredient in these four-dimensional counterterms are linear couplings of the abelian field strength to the various two-forms arising from reduction of $B^{(2)}$ and $B^{(6)}$ on a basis of internal two-cycles ω_k and their dual four-cycles $\hat{\omega}_k$ as $B^{(2)} = b_0^{(2)} + \ell_s^2 \sum_{k=1}^{h_{11}} b_k^{(0)} \omega_k$, $B^{(6)} = \ell_s^6 b_0^{(0)} \text{vol}_6 + \ell_s^4 \sum_{k=1}^{h_{11}} b_k^{(2)} \hat{\omega}_k$. Collecting all contributions of this type one finds

$$S_{mass} = \sum_x \sum_{k=0}^{h_{11}} \frac{Q_k^x}{2\pi\alpha'} \int_{\mathbb{R}_{1,3}} f_x \wedge b_k^{(2)}, \quad (7)$$

where the abelian field strengths are collectively denoted by f_x and for brevity we refer again to [4, 5] for the concrete formulae for Q in both heterotic theories. The couplings (7) induce a mass for every linear combination of $U(1)$ s which does not lie in the kernel of the mass matrix Q . Let us point out that all mass terms are of the same order in both string and sigma model perturbation theory. The number of massive $U(1)$ s will always be at least as big as the number of anomalous $U(1)$ s. Though all entries in the mass matrix are of order M_s^2 , the mass eigenstates of the gauge bosons can have masses significantly lower than the string scale.

4 π -stability for supersymmetry at one loop

Supersymmetry at string tree-level imposes the well-known constraint $g^{ab}F_{ab} = 0$ on the field strength F of W , or equivalently that F be holomorphic, $F^{(2,0)} = 0 = F^{(0,2)}$, and satisfy $J \wedge J \wedge F = 0$. The Donaldson-Uhlenbeck-Yau theorem translates this latter condition, the zero-slope limit of the Hermitian Yang-Mills equation, into the requirement that each summand bundle V_i be μ -stable and satisfy the integrability condition $\int_{\mathcal{M}} J \wedge J \wedge c_1(V_i) = 0$, which is of course trivially fulfilled for gauge bundles with vanishing first Chern class. Being a non-holomorphic supersymmetry constraint, this so-called DUY condition is naturally expected to arise as a D-term in the four-dimensional effective supergravity and is therefore, at the string perturbative level, subject to at most one-loop corrections. Fortunately, we actually have just encountered important one-loop terms in our effective action by standard anomaly considerations, the mass couplings analysed in the previous section. They carry the relevant information to determine the potential one-loop corrections to the DUY equation. The key point is to notice that the linear mass couplings involve the axionic partners complexifying the dilaton and Kähler moduli as $S = \frac{1}{2\pi} \left[e^{-2\phi_{10}} \frac{\text{Vol}(\mathcal{M})}{\ell_s^6} + i b_0^{(0)} \right]$ and $T_k = \frac{1}{2\pi} \left[-\alpha_k + i b_k^{(0)} \right]$. To maintain gauge invariance in the Kähler potential \mathcal{K} of the effective $\mathcal{N} = 1$ supergravity, the couplings (7) also enforce an appropriate modification of \mathcal{K} as

$$\begin{aligned} \mathcal{K} = & \frac{M_{pl}^2}{8\pi} \left[-\ln \left(S + S^* - \sum_x Q_0^x V_x \right) - \ln \left(- \sum_{i,j,k=1}^{h_{11}} \frac{d_{ijk}}{6} \left(T_i + T_i^* - \sum_x Q_i^x V_x \right) \right. \right. \\ & \left. \left. \left(T_j + T_j^* - \sum_x Q_j^x V_x \right) \left(T_k + T_k^* - \sum_x Q_k^x V_x \right) \right) \right], \end{aligned} \quad (8)$$

where by V_x we denote the abelian superfields. As a standard matter of fact, the $\mathcal{N} = 1$ Kähler potential is related to the Fayet-Iliopoulos D-terms terms via the relation $\frac{\xi_x}{g_x^2} = \frac{\partial \mathcal{K}}{\partial V_x} \Big|_{V=0}$. We therefore arrive at the

following tree-level and one-loop contributions to the FI terms

$$\frac{\xi_x}{g_x^2} = -\frac{e^{2\phi_{10}} M_{pl}^2 \ell_s^6}{4\text{Vol}(\mathcal{M})} \left(\frac{1}{4} e^{-2\phi_{10}} \sum_{i,j,k=1}^{h_{11}} d_{ijk} Q_i^x \alpha_j \alpha_k - \frac{1}{2} Q_0^x \right). \quad (9)$$

Inserting the concrete expressions for the charges we find that the FI terms vanish if and only if⁵

$$\begin{aligned} \int_{\mathcal{M}} J \wedge J \wedge c_1(L_n) - \frac{1}{2} g_s^2 \ell_s^4 \int_{\mathcal{M}} c_1(L_n) \wedge \left(\sum_{i=1}^K \text{ch}_2(V_i) + \sum_{m=1}^M a_m c_1^2(L_m) + \frac{1}{2} c_2(T) \right) &= 0, \\ \int_{\mathcal{M}} J \wedge J \wedge c_1(V_i) - 2 g_s^2 \ell_s^4 \int_{\mathcal{M}} (\text{ch}_3(V_i) + \frac{1}{24} c_1(V_i) c_2(T)) &= 0 \end{aligned} \quad (10)$$

for the $E_8 \times E_8$ and $SO(32)$ case, respectively. Note that in the first case, the one-loop term contains a sum over all bundles in the same E_8 factor as L_n , whereas for the $SO(32)$ string it is "local" in that it only depends on the bundle V_i under consideration. Since the tree-level part of this expression constitutes the conventional DUY equation, we interpret (10) as the one-loop corrected integrability condition for the local supersymmetry equation. Arising as a D-term, it poses constraints on a particular combination of the dilaton and the Kähler moduli. For consistency these values have to lie in the perturbative regime and inside the Kähler cone. As recalled above, to be also *sufficient* for supersymmetry, the DUY equation has to be supplemented by an appropriate stability condition. In fact, we argued in [5] that the modified stability condition to be satisfied by each subbundle in addition to (10) is precisely that of π -stability [8]. For practical applications it is satisfactory to note that, at least in the perturbative regime, μ -stable bundles are also π -stable [8], but since the converse is not true it would be interesting to investigate the moduli space of bundles acceptable only under the latter notion of stability. Let us simply state here that a further relevant effect of the anomaly counterterms is to generate non-universal one-loop corrections to the gauge kinetic functions [4, 5]. For consistency of the supersymmetric framework we have to ensure that their real part is still positive, i.e.

$$\begin{aligned} \int_{\mathcal{M}} J \wedge J \wedge J - \frac{3}{2} g_s^2 \ell_s^4 \int_{\mathcal{M}} J \wedge \left(\frac{4}{3} a_n c_1^2(L_n) + \sum_{i=1}^K \text{ch}_2(V_i) + \sum_{m=1}^M a_m c_1^2(L_m) + \frac{1}{2} c_2(T) \right) &> 0, \\ \frac{n}{3!} \int_{\mathcal{M}} J \wedge J \wedge J - g_s^2 \ell_s^4 \int_{\mathcal{M}} J \wedge (\text{ch}_2(V_i) + \frac{n}{24} c_2(T)) &> 0, \end{aligned} \quad (11)$$

respectively⁶. One can then verify that S-duality translates the complete supersymmetry constraint of the $SO(32)$ theory precisely into the perturbative part of the celebrated Π -stability condition for spacetime-filling D-branes. Of course, (10) and (11) may receive non-perturbative corrections in α' and g_s . In view of S-duality, it is therefore desirable to investigate the possible analogue of full Π -stability for both heterotic theories.

5 A Standard-Model like example on an elliptically fibered Calabi-Yau

As a demonstration of the new model building possibilities let us present a simple example with Standard Model-like gauge group and chiral matter in the $SO(32)$ string. As has become clear, the general strategy is to construct *stable holomorphic* vector bundles with known topological invariants on a given Calabi-Yau and to ensure that the various consistency conditions (5,6,10,11) are satisfied. It is then possible to engineer systematically interesting semi-realistic low-energy properties. Fortunately, the so-called spectral cover construction provides us with a large class of such stable $SU(n)$ bundles on elliptically fibered Calabi-Yaus. These bundles can then be further twisted by line bundles to yield structure group $U(n)$ as needed for our $SO(32)$ models. We refer the reader to [6] for the details of the spectral cover construction; here we can merely recall the main ingredients. Consider an elliptically fibered Calabi-Yau threefold \mathcal{M} with projection $\pi : \mathcal{M} \rightarrow B$ and a section $\sigma : B \rightarrow \mathcal{M}$ which identifies the base B as a submanifold of

⁵ We assume here that the VEVs of the matter fields charged under the abelian gauge group are zero.

⁶ The first inequality has to be satisfied by each line bundle L_n in the $E_8 \times E_8$ models; unlike with (10), the analogous condition on the V_i is non-trivial and simply given by omitting the term $\frac{4}{3} a_n c_1^2(L_n)$.

\mathcal{M} . If the base is smooth and preserves only $\mathcal{N} = 1$ supersymmetry in four dimensions, it is restricted to a del Pezzo surface, a Hirzebruch surface, an Enriques surface or a blow up of a Hirzebruch surface. The idea is to use a simple description of $SU(n)$ bundles over the elliptic fibers E_b over each base point b and then globally glue them together to define bundles over \mathcal{M} . One of the many nice properties of such a construction is that eventually the Chern classes of such bundles can be computed entirely in terms of data defined on the base B . The $SU(n)$ bundle is specified partly by the spectral cover C , an n -fold cover of B with $\pi_C : C \rightarrow B$. One has the freedom to further twist it by η , the pull-back of a line bundle on the base. This determines the cohomology class of C as $[C] = n\sigma + \eta$. In addition one has to choose a line bundle \mathcal{N} on C defined such that $V|_B = \pi_C^* \mathcal{N}$. The vector bundle V is then given as $V = \pi_{1*}(\pi_2^* \mathcal{N} \otimes \mathcal{P})$, where π_1 and π_2 denote the two projections of the fiber product $Y = \mathcal{M} \times_B C$ onto the two factors \mathcal{M} and C and \mathcal{P} represents the Poincaré bundle on Y . If the spectral cover is irreducible, i.e. if the linear system $|\eta|$ is basepoint-free and $\eta - n c_1(B)$ is effective [7], then the in general semi- μ -stable bundle V is truly stable. To arrive at a $U(n)$ bundle, we twist V by an arbitrary line bundle \mathcal{Q} on \mathcal{M} with $c_1(\mathcal{Q}) = q\sigma + c_1(\zeta)$ to get $V_{\mathcal{Q}} = V \otimes \mathcal{Q}$. Note that the process of twisting does not affect the stability properties of the bundle. For the very explicit expressions of the Chern classes of $V_{\mathcal{Q}}$ in terms of the above data please consult [2]. Suffice it here to state that they are determined entirely by the rank n of V , $c_1(\mathcal{Q})$, η and a further rational number λ which has to be chosen appropriately to guarantee integer expressions for the Chern classes.

Let us now outline a concrete example in the framework of the $SO(32)$ theory on an elliptic fibration over the del Pezzo surface $B = dP_4$. More details on the computations can be found in [2]. The second cohomology class of dP_4 is generated by the elements l, E_1, \dots, E_4 with intersection form $l \cdot l = 1, l \cdot E_m = 0, E_m \cdot E_n = -\delta_{m,n}$. Being interested in Standard-like models we aim at obtaining a visible gauge group $U(3)_{\alpha} \times U(2)_{\beta} \times U(1)_{\gamma} \times U(1)_{\delta}$ and at realizing the quarks and leptons as appropriate bifundamentals. A possible choice of the hypercharge as a (massless) combination of the abelian factors is given by $Q_Y = \frac{1}{6}Q_{\alpha} + \frac{1}{2}(Q_{\gamma} + Q_{\delta})$. In this case, also some of the (anti-)symmetric representations carry MSSM quantum numbers. The details of the chiral MSSM spectrum we try to reproduce can be found in Table 2. Among the many possibilities we consider the simple embedding of the structure group $G = U(1)_a \times U(1)_b \times U(2)_c \times U(1)_d$ into $U(3) \times U(2) \times U(2) \times U(1)$. This breaks $SO(32)$ to the commutator $U(3)_{\alpha} \times U(2)_{\beta} \times U(1)_{\gamma} \times U(1)_{\delta} \times SO(16)$ modulo the issue of anomalous abelian factors. The abelian bundles are defined by $c_1(V_a) = \sigma + 5l - 3E_1 - 5E_2 - E_3 = -c_1(V_d), c_1(V_b) = \sigma + 5l - 3E_1 - 5E_2 - E_3$ and V_c is specified by $\eta_c = 7l - 3E_1 - 3E_2 - E_3 - E_4$ and $q_c = 0 = \zeta_c$.⁷

One may verify explicitly that the conditions on η_c for μ -stability are satisfied. Let us also point out that the configuration is free of the Witten anomaly (cf. (5)). Furthermore, the $U(1)_Y$ hypercharge is indeed massless as desired. However, since the rank of the mass matrix is two, we get another massless $U(1)$ in the four-dimensional gauge group, which is identified as $U(1)_c$. The perturbative low energy gauge group is therefore

$$H = SU(3) \times SU(2) \times U(1)_Y \times U(1)' \times SO(16). \quad (10)$$

The degeneracy of the bundle V_a and $V_d = V_a^*$ leads to a gauge enhancement of the $U(3)_a$ and the $U(1)_d$ to a $U(4)$. Apart from these drawbacks, the configuration indeed gives rise to three families of the MSSM chiral spectrum as listed in Table 2.

In addition, we find some chiral exotic matter in the antisymmetric representation of the $U(2)$ and in the bifundamental of the $SO(16)$ with the $U(3)$ and $U(2)$, respectively. The chosen bundles alone do not satisfy the tadpole cancellation condition. However, the resulting tadpole class is effective, i.e. corresponds to the class of a veritable curve on \mathcal{M} , and can therefore be cancelled by including H5-branes. This demonstrates the importance of allowing for these non-perturbative objects. We therefore find an additional symplectic gauge group of rank 74 including chiral bifundamental matter. Let us conclude this example by stating that the DUY equations can be satisfied for Kähler moduli inside the Kähler cone and in the perturbative regime, together with positivity of the real part of the various gauge kinetic functions.

⁷ Note that the fact that we are actually considering an $SU(2)$ instead of a $U(2)$ bundle is an artifact of this particular model and makes no difference in the group theoretic decomposition of $SO(32)$.

$U(3)_\alpha \times U(2)_\beta \times U(1)_\gamma \times U(1)_\delta \times SO(16) \times \prod_a Sp(2N_a)$				
MSSM particle	repr.	index	mult.	total
Q_L	$(\mathbf{3}, \bar{\mathbf{2}}; 1, 1)_{(1, -1, 0, 0)}$	$\chi(X, V_a \otimes V_b^*)$	8	
Q_L	$(\mathbf{3}, \mathbf{2}; 1, 1)_{(1, 1, 0, 0)}$	$\chi(X, V_a \otimes V_b)$	-11	-3
u_R	$(\bar{\mathbf{3}}, 1; 1, 1)_{(-1, 0, -1, 0)}$	$\chi(X, V_a^* \otimes V_c^*)$	-3	
u_R	$(\bar{\mathbf{3}}, 1; 1, 1)_{(-1, 0, 0, -1)}$	$\chi(X, V_a^* \otimes V_d^*)$	0	-3
d_R	$(\bar{\mathbf{3}}, 1; 1, 1)_{(-1, 0, 1, 0)}$	$\chi(X, V_a^* \otimes V_c)$	-3	
d_R	$(\bar{\mathbf{3}}, 1; 1, 1)_{(-1, 0, 0, 1)}$	$\chi(X, V_a^* \otimes V_d)$	45	
d_R	$(\bar{\mathbf{3}}_A, 1; 1, 1)_{(2, 0, 0, 0)}$	$\chi(X, \bigotimes_s^2 V_a)$	-45	-3
L	$(1, \mathbf{2}; 1, 1)_{(0, 1, -1, 0)}$	$\chi(X, V_b \otimes V_c^*)$	-7	
L	$(1, \mathbf{2}; 1, 1)_{(0, 1, 0, -1)}$	$\chi(X, V_b \otimes V_d^*)$	-11	
L	$(1, \bar{\mathbf{2}}; 1, 1)_{(0, -1, -1, 0)}$	$\chi(X, V_b^* \otimes V_c^*)$	7	
L	$(1, \bar{\mathbf{2}}; 1, 1)_{(0, -1, 0, -1)}$	$\chi(X, V_b^* \otimes V_d^*)$	8	-3
e_R	$(1, 1; 1, 1)_{(0, 0, 2, 0)}$	$\chi(X, \bigwedge^2 V_c)$	0	
e_R	$(1, 1; 1, 1)_{(0, 0, 0, 2)}$	$\chi(X, \bigwedge^2 V_d)$	0	
e_R	$(1, 1; 1, 1)_{(0, 0, 1, 1)}$	$\chi(X, V_c \otimes V_d)$	-3	-3
ν_R	$(1, 1; 1, 1)_{(0, 0, -1, 1)}$	$\chi(X, V_c^* \otimes V_d)$	-3	-3

Table 2 Chiral MSSM spectrum for a four-stack model with $Q_Y = \frac{1}{6}Q_\alpha + \frac{1}{2}(Q_\gamma + Q_\delta)$.

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